

# General bounds on non-standard neutrino interactions

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## Abstract

We derive model-independent bounds on production and detection non-standard neutrino interactions (NSI). We find that the constraints for NSI parameters are around  $\mathcal{O}(10^{-2})$  to  $\mathcal{O}(10^{-1})$ . Furthermore, we review and update the constraints on matter NSI. We conclude that the bounds on production and detection NSI are generally one order of magnitude stronger than their matter counterparts.

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## I. INTRODUCTION

The formalism of non-standard neutrino interactions (NSI) is a very widespread and convenient way of parametrising the effects of new physics in neutrino oscillations [1, 2, 3, 4, 5, 6, 7, 8, 9]. Even though present data constrain NSI to be a subleading effect in neutrino oscillation experiments, the possibility of their eventual detection or interference with neutrino oscillations at present [10, 11, 12, 13, 14, 15, 16, 17] and future [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31] experiments has triggered a considerable interest in the community. In particular, it is a common practice to study NSI in matter, which correspond to neutral-current-like operators, assuming that the constraints on the NSI affecting production and detection processes are much stronger. However, up to now, only model-dependent bounds on such interactions are present in the literature [10, 18]. The main aim of this paper is filling this gap and providing model-independent bounds on NSI affecting neutrino production and detection processes. Also, the present constraints on matter NSI will be reviewed and updated.

Before entering into details and deriving the current bounds, a discussion on the naturalness of large NSI is in order. In particular, this argument has been faced for matter NSI [32, 33, 34], but the main message can be applied to production and detection NSI as well. Matter NSI are defined through the following addition to the Lagrangian density:

$$\mathcal{L}_{\text{NSI}}^M = -2\sqrt{2}G_F\varepsilon_{\alpha\beta}^{fP}[\bar{f}\gamma^\mu Pf][\bar{\nu}_\alpha\gamma_\mu P_L\nu_\beta], \quad (1)$$

where  $f = e, u, d$  and  $\varepsilon_{\alpha\beta}^{fP}$  encodes the deviation from standard interactions. For example, an operator of this kind is induced in fermionic seesaw models once the heavy fermions (singlets or triplets) are integrated out leading to a  $d = 6$  operator that modifies the neutrino kinetic energy [35, 36, 37, 38]. After a transformation to obtain canonical kinetic terms, modified couplings of the leptons to the gauge bosons, characterized by deviations from unitarity of the leptonic mixing matrix, are induced. Upon integrating out the gauge bosons with their modified couplings, NSI operators are therefore obtained. Because of the strong bounds on the unitarity of this matrix, these NSI are constrained to be  $\lesssim \mathcal{O}(10^{-3})$  [32, 37, 39]. This means that their eventual detection is challenging, although not impossible, at future facilities [40, 41, 42, 43, 44].

On the other hand, large NSI could be generated by some other new physics, not necessarily related to neutrino masses, at an energy above the electroweak scale. As a consequence,

an  $SU(2)$  gauge invariant formulation of NSI is mandatory. The simplest gauge invariant realization of the operator in Eq. (1) implies to promote the neutrino fields to full lepton doublets. However, in that case, strong bounds stemming from four-charged-fermion processes would apply [45, 46, 47]. In order to avoid these constraints, cancellations among different higher-dimensional operators are required [33, 45]. In the case of  $d = 6$  operators there is only one combination which satisfies these conditions and the corresponding NSI are also severely constrained [32, 48]. In the case of  $d = 8$  operators it has been shown that, avoiding cancellations between diagrams involving different messenger fields or the introduction of new leptonic doublets that could dangerously affect the electroweak precision tests, the only possibilities of evading the constraints imposed by gauge invariance reduce to the cases already mentioned, with the consequent stringent bounds [32]. Therefore, in order to realise the cancellations that would allow large NSI, some fine-tuning is needed. An example of the naturalness prize required is presented in the toy model proposed in Ref. [33]. However, even if large NSI are generated in this way at tree-level, dangerous quadratic divergences contributing to four-charged-fermion operators appear at one-loop [34]. In order to have large NSI, another fine-tuning would then be required at one-loop unless the scale of new physics is smaller than  $4\pi v$ , where  $v$  is the Higgs vev. Alternatively, a symmetry could guarantee the cancellation both at tree- and loop-level, but so far no model has been found with these characteristics, *i.e.*, leading to large NSI.

From the previous discussion it is clear that it is not easy to induce large neutrino NSI in a specific theoretical framework. However, since it is impossible to exclude them completely in a model-independent way and since their effects may be visible at future experiments, we think it is worthwhile to derive their present bounds.

The rest of the paper is organized as follows: First, in Section II, we define charged-current-like NSI and derive bounds on various combinations of  $\varepsilon$ , specifying which bounds can be set if only one non-zero  $\varepsilon$  is considered at a time. We then proceed by discussing loop bounds on charged-current-like NSI in Section III. Finally, we review and update the bounds on matter NSI in Section IV and make a summary of the results and conclude in Section V.

## II. CHARGED-CURRENT-LIKE NON-STANDARD INTERACTIONS

Let us start by considering NSI for source and detector processes. Since these are always based on charged-current processes so as to tag the neutrino flavour through the flavour of the associated charged lepton, we will refer to these as charged-current-like NSI. The general leptonic NSI are given by the effective Lagrange density

$$\mathcal{L}_{\text{NSI}}^\ell = -2\sqrt{2}G_F\varepsilon_{\gamma\delta}^{\alpha\beta P}[\bar{\ell}_\alpha\gamma^\mu P\ell_\beta][\bar{\nu}_\gamma\gamma_\mu P_L\nu_\delta], \quad (2)$$

where  $P$  is either  $P_L$  or  $P_R$  and, due to Hermiticity,  $\varepsilon_{\gamma\delta}^{\alpha\beta P} = \varepsilon_{\delta\gamma}^{\beta\alpha P*}$ . For charged-current-like NSI  $\alpha \neq \beta$ ; in particular,  $\alpha = \mu$  and  $\beta = e$  are the only parameters of importance for neutrino oscillation experiments due to their effect in neutrino production via muon decay. Notice that  $\alpha = \beta = e$  would instead correspond to matter NSI.

In a similar fashion, the charged-current-like NSI with quarks are given by the effective Lagrange density

$$\mathcal{L}_{\text{NSI}}^q = -2\sqrt{2}G_F\varepsilon_{\alpha\beta}^{qq'P}V_{qq'}[\bar{q}\gamma^\mu Pq'][\bar{\ell}_\alpha\gamma_\mu P_L\nu_\beta] + \text{h.c.}, \quad (3)$$

where  $q$  is an up-type and  $q'$  is a down-type quark. Naturally, only  $q = u$  and  $q' = d$  are of practical interest for neutrino oscillations, due to their contributions to charged-current interactions with pions and nuclei. Because of this, we will concentrate on constraining  $\varepsilon_{\alpha\beta}^{\mu e}$  as well as  $\varepsilon_{\alpha\beta}^{ud}$ . Since the relevant combinations of NSI that contribute to some processes will be of an axial or vector structure we define

$$\varepsilon_{\alpha\beta}^{\gamma\delta V} = \varepsilon_{\alpha\beta}^{\gamma\delta R} + \varepsilon_{\alpha\beta}^{\gamma\delta L}, \quad (4)$$

$$\varepsilon_{\alpha\beta}^{\gamma\delta A} = \varepsilon_{\alpha\beta}^{\gamma\delta R} - \varepsilon_{\alpha\beta}^{\gamma\delta L}, \quad (5)$$

in order to simplify the notation. Notice that more general Dirac structures such as scalar or tensor couplings can in principle be considered to generalise Eqs. (2) and (3). However, these NSI will have the wrong chirality to contribute coherently with the SM production and detection processes –something that is usually assumed for NSI– and therefore linear interference of these NSI will require an extra chirality suppression [25, 49]. For this reason we will neglect them here.

### A. Bounds from kinematic Fermi constant

At present, the most precise determination of the Fermi constant  $G_F$  is through the muon decay rate. However, if NSI of the form  $\varepsilon_{\alpha\beta}^{\mu e}$  are present, this will make the measured Fermi constant from muon decays  $G_\mu$  differ from the true Fermi constant according to the relation  $G_\mu = G_F f(\varepsilon_{e\mu}^{\mu e L}, \sum_{\alpha\beta P} |\varepsilon_{\alpha\beta}^{\mu e P}|^2)$ . Here we have introduced the function

$$f(x, y) = 1 + 2 \operatorname{Re}(x) + y, \quad (6)$$

where  $x$  represents the interference between the SM and the particular NSI that contributes coherently with the SM to the process and  $y$  is the incoherent sum of the NSI contributions. Therefore, in all the processes considered, stronger bounds will be implied for the real part of  $x$ . Given the relation between  $G_\mu$  and  $G_F$ , an independent measurement of the Fermi constant will constrain  $\varepsilon_{\alpha\beta}^{\mu e P}$ . We will consider two different ways of deriving the value of  $G_F$ , one involving only the kinematic measurements of the gauge boson masses and one involving comparison to the quark sector.

For determining  $G_F$  from kinematic considerations, we need to review the predictions of the Standard Model. From Ref. [50], we have

$$M_W = \frac{A_0}{s_W \sqrt{1 - \Delta r}}, \quad (7)$$

where  $A_0 = \sqrt{\pi\alpha/(\sqrt{2}G_F)}$ ,  $s_W^2 = 1 - M_W^2/M_Z^2$ ,  $\alpha$  is the fine-structure constant, and  $\Delta r = 0.03690 \pm 0.0007$  is the radiative correction to the tree-level relation. Thus, we obtain the relation

$$G_F = \frac{\pi\alpha M_Z^2}{\sqrt{2}M_W^2(M_Z^2 - M_W^2)(1 - \Delta r)}. \quad (8)$$

For the masses of the vector bosons, we use the combined fit for the  $W$  mass from LEP and Tevatron,  $M_W = 80.398 \pm 0.025$  GeV, as well as  $M_Z = 91.1876 \pm 0.0021$  GeV from LEP [50]. The resulting Fermi coupling constant is

$$G_F = (1.1696 \pm 0.0020) \cdot 10^{-5} \text{ GeV}^{-2} \quad (1\sigma). \quad (9)$$

Comparing with  $G_\mu$ , we obtain

$$\frac{G_\mu}{G_F} = f(\varepsilon_{e\mu}^{\mu e L}, \sum_{\alpha\beta P} |\varepsilon_{\alpha\beta}^{\mu e P}|^2) = \frac{1.16637 \pm 0.00001}{1.1696 \pm 0.0020} = 0.9973 \pm 0.0017, \quad (10)$$

which represents a 90 % confidence level agreement with the Standard Model expectation. The only truly model-independent bound that we can extract from this is on the combination  $f(\varepsilon_{e\mu}^{\mu e L}, \sum_{\alpha\beta P} |\varepsilon_{\alpha\beta}^{\mu e P}|^2)$ . On the other hand, it is common practice to assume the presence of only one non-zero  $\varepsilon$  at a time in order to avoid cancellations inside  $f(\varepsilon_{e\mu}^{\mu e L}, \sum_{\alpha\beta P} |\varepsilon_{\alpha\beta}^{\mu e P}|^2)$ . In this way, the following bounds can be obtained:<sup>1</sup>

$$\text{Re}(\varepsilon_{e\mu}^{\mu e L}) = (-1.4 \pm 1.4) \cdot 10^{-3}, \quad (11)$$

$$|\varepsilon_{\alpha\beta}^{\mu e P}| < 0.030, \quad (12)$$

at 90 % confidence level.

## B. Bounds from CKM unitarity

One way of constraining the completely leptonic NSI, as well as some of the charged-current NSI with quarks, is to make the assumption that the Cabibbo–Kobayashi–Maskawa (CKM) matrix is unitary, as predicted by the Standard Model. The experimental test of the CKM unitarity is essentially based upon the determination of  $V_{ud}$  and  $V_{us}$  from beta- and Kaon-decays,<sup>2</sup> where the Fermi constant extracted from muon decay  $G_\mu$  is used to predict the decay rates. These are proportional to

$$\Gamma \propto G_F^2 |V_{ux}|^2, \quad (13)$$

which means that, by inserting  $G_\mu$  in place of  $G_F$ , we are actually determining  $|V_{ux}^M|^2 \equiv |V_{ux}|^2 / f^2(\varepsilon_{e\mu}^{\mu e L}, \sum_{\alpha\beta P} |\varepsilon_{\alpha\beta}^{\mu e P}|^2)$ . Adding the information from beta- and Kaon-decay experiments and assuming that leptonic NSI dominate over quark NSI, we have [50]

$$|V_{ud}^M|^2 + |V_{us}^M|^2 = \frac{|V_{ud}|^2 + |V_{us}|^2}{f^2(\varepsilon_{e\mu}^{\mu e L}, \sum_{\alpha\beta P} |\varepsilon_{\alpha\beta}^{\mu e P}|^2)} = \frac{1}{f^2(\varepsilon_{e\mu}^{\mu e L}, \sum_{\alpha\beta P} |\varepsilon_{\alpha\beta}^{\mu e P}|^2)} = 0.9999 \pm 0.0010 \quad (14)$$

at  $1\sigma$ , where the CKM unitarity is inserted in the second step. Again, this translates into a bound for  $f(\varepsilon_{e\mu}^{\mu e L}, \sum_{\alpha\beta P} |\varepsilon_{\alpha\beta}^{\mu e P}|^2)$ , but making the assumption of having only one non-zero  $\varepsilon$  at a time we obtain:

$$|\text{Re}(\varepsilon_{e\mu}^{\mu e L})| < 4.0 \cdot 10^{-4}, \quad (15)$$

<sup>1</sup> Throughout the paper we will follow the statistical approach proposed in Ref. [51] by Feldman and Cousins.

<sup>2</sup> In principle  $V_{ub}$  should also be considered. However, its value is smaller than the uncertainty in the other two matrix elements and we therefore leave it out of our discussion.

$$|\varepsilon_{\alpha\beta}^{\mu e P}| < 0.030, \quad (16)$$

at the 90 % confidence level. Notice that the bound of Eq. (15) is slightly stronger than the one obtained from the kinematic determination of the Fermi constant, but it relies on one extra assumption, *i.e.*, the unitarity of the CKM matrix.

On the other hand, if we assume that the NSI with quarks are dominating, then the insertion of  $G_\mu$  in place of  $G_F$  is not leading to any ambiguities. However, NSI of the form  $\varepsilon^{ud}$  will contribute to the beta-decay rate, through which  $V_{ud}$  is extracted. Experimentally, only superallowed  $0^+ \rightarrow 0^+$  decays are considered, which means that the nuclear matrix element will have a vector structure and, therefore, only the vector NSI combination will contribute in the following way:

$$\Gamma_\beta \propto G_F^2 |V_{ud}|^2 f(\varepsilon_{ee}^{udV}, \sum_\alpha |\varepsilon_{e\alpha}^{udV}|^2). \quad (17)$$

Since the Kaon decays are not affected by  $\varepsilon^{ud}$ , these can be used to extract  $V_{ud}$  indirectly from the assumption of CKM unitarity (*i.e.*,  $|V_{ud}|^2 = 1 - |V_{us}|^2$ ). The result of this operation is  $|V_{ud}|^2 = 0.94915 \pm 0.00086$  [50], which should be compared to the value of  $|\tilde{V}_{ud}|^2 = |V_{ud}|^2 f(\varepsilon_{ee}^{udV}, \sum_\alpha |\varepsilon_{e\alpha}^{udV}|^2)$  derived from beta decays  $|\tilde{V}_{ud}|^2 = 0.94903 \pm 0.00055$  [50]. Once again a truly model-independent bound can only be extracted for the combination  $f(\varepsilon_{ee}^{udV}, \sum_\alpha |\varepsilon_{e\alpha}^{udV}|^2)$ , but making the assumption of taking one  $\varepsilon$  at a time we obtain:

$$|\text{Re}(\varepsilon_{ee}^{udV})| < 0.00086, \quad (18)$$

$$|\varepsilon_{e\alpha}^{udV}| < 0.041. \quad (19)$$

Notice that, unlike the determination through the kinematic  $G_F$ , the determination of the non-standard parameters  $\varepsilon_{\alpha\beta}^{\mu e}$  through CKM unitarity relies on the assumption that the quark interactions are not affected, making the resulting bounds slightly more model-dependent. On the other hand, if a given model predicts both lepton ( $\varepsilon_{\alpha\beta}^{\mu e}$ ) and quark ( $\varepsilon_{\alpha\beta}^{ud}$ ) NSI simultaneously, the bounds on  $\varepsilon_{\alpha\beta}^{\mu e}$  from the kinematic  $G_F$  compared to muon decay would still apply, while somewhat weaker bounds on  $\varepsilon_{\alpha\beta}^{ud}$  could still be derived after propagating the errors derived on the former through the CKM unitarity relation.

### C. Bounds from pion processes

For the quark charged-current NSI involving charged leptons other than electrons, the universality tests stemming from the relative decay rates of charged pions as well as that of taus into pions can be used to set bounds. The squared and summed matrix element involving a charged pion, a charged lepton and a neutrino is modified according to

$$\sum_{\beta} |\mathcal{M}(\pi, \ell_{\alpha}, \nu_{\beta})|^2 = |\mathcal{M}(\pi, \ell_{\alpha}, \nu_{\alpha})|^2 f(\varepsilon_{\alpha\alpha}^{udA}, \sum_{\beta} |\varepsilon_{\alpha\beta}^{udA}|^2). \quad (20)$$

This modification is equivalent to violations of weak interaction flavor universality identifying  $g^2 f(\varepsilon_{\alpha\alpha}^{udA}, \sum_{\beta} |\varepsilon_{\alpha\beta}^{udA}|^2) = g_{\alpha}^2$ , where  $g_{\alpha}$  is the  $W$  coupling to the lepton flavour  $\alpha$ . Comparing the rates of  $\pi \rightarrow e\nu$ ,  $\pi \rightarrow \mu\nu$  and  $\tau \rightarrow \pi\nu$ , bounds can be set on the ratios  $g_{\alpha}/g_{\beta}$ . From Ref. [52] we have

$$\frac{g_{\mu}}{g_e} = 1.0021 \pm 0.0016 \quad \text{and} \quad \frac{g_{\tau}}{g_{\mu}} = 1.0030 \pm 0.0034 \quad (21)$$

at  $1\sigma$ . Thus, if only one  $\varepsilon$  is considered at a time, we obtain the following bounds at the 90 % confidence level:

$$\text{Re}(\varepsilon_{\mu\mu}^{udA}) = (2.1 \pm 2.6) \cdot 10^{-3}, \quad (22)$$

$$|\varepsilon_{\mu\alpha}^{udA}| < 0.078, \quad (23)$$

$$\text{Re}(\varepsilon_{\tau\tau}^{udA}) = (3.0 \pm 5.5) \cdot 10^{-3}, \quad (24)$$

$$|\varepsilon_{\tau\alpha}^{udA}| < 0.13, \quad (25)$$

$$\text{Re}(\varepsilon_{ee}^{udA}) = (-2.1 \pm 2.6) \cdot 10^{-3}, \quad (26)$$

$$|\varepsilon_{e\alpha}^{udA}| < 0.045. \quad (27)$$

Notice that the bounds on  $|\varepsilon_{e\alpha}^{udA}|$  are more stringent than the bounds on  $|\varepsilon_{\mu\alpha}^{udA}|$  because the offset of the best-fit from the Standard Model expectation goes in the opposite direction with respect to the effect of  $|\varepsilon_{e\alpha}^{udA}|$ .

It is important to note that a model that predicts equal  $f(\varepsilon_{\alpha\alpha}^{udA}, \sum_{\beta} |\varepsilon_{\alpha\beta}^{udA}|^2)$  for  $\alpha = e, \mu, \tau$  cannot be bounded using this type of argument, since it affects all of these decays in the same way and universality is not violated. However, if we only consider  $\varepsilon$  of one chirality at a time, then this would imply that  $\varepsilon_{\mu\alpha}^{udP}$  and  $\varepsilon_{\tau\alpha}^{udP}$  share the stronger bounds derived for  $\varepsilon_{e\alpha}^{udP}$  from the CKM unitarity.



In a similar fashion, we can use the universality test between the  $\mu \rightarrow e\nu\bar{\nu}$  and  $\tau \rightarrow \mu\nu\bar{\nu}$  decays to constrain the non-standard couplings  $\varepsilon_{\alpha\beta}^{\mu e}$ . This constraint is related to the lepton universality ratio  $g_\tau/g_e = 1.0004 \pm 0.0022$ . Therefore, the inverse of this number is a measurement of  $\sqrt{f(\varepsilon_{e\mu}^{\mu e L}, \sum_{\alpha\beta P} |\varepsilon_{\alpha\beta}^{\mu e P}|^2)}$ , where we disregard possible modifications of the tau decay which are not important for neutrino oscillation experiments. The resulting bounds are:

$$\text{Re}(\varepsilon_{e\mu}^{\mu e}) = (-0.4 \pm 3.5) \cdot 10^{-3}, \quad (28)$$

$$|\varepsilon_{\alpha\beta}^{\mu e}| < 0.080. \quad (29)$$

#### D. Bounds from oscillation experiments

Production and detection NSI imply that a neutrino produced or detected in association with a charged lepton will not necessarily share its flavour. This means that flavour conversion is present already at the interaction level and “oscillations” can occur at zero distance. Indeed, in the presence of NSI,

$$P_{\alpha\beta}(L=0) \simeq |\varepsilon_{\alpha\beta}^{udA}|^2 \quad (30)$$

if the neutrino is produced through pion decays and

$$P_{e\alpha}(L=0) \simeq \sum_{\beta P} |\varepsilon_{\alpha\beta}^{\mu e P}|^2 \quad \text{as well as} \quad P_{\mu\beta}(L=0) \simeq \sum_{\alpha P} |\varepsilon_{\alpha\beta}^{\mu e P}|^2 \quad (31)$$

for neutrinos produced through muon decays. For the detection through inverse beta decays the situation is a bit more involved since the relative contributions of the different chiralities vary depending on the energy regime due to the nuclear matrix elements. Here we will discuss the cases of very low ( $E < 1$  GeV) and very high ( $E > 10$  GeV) energies. In the first case the neutrino-nucleon cross section is proportional to  $(g_V^2 + 3g_A^2)$ , where  $g_V = 1$  and  $g_A = 1.23$ . This means that the vector and axial combinations of the NSI that can mediate the processes will contribute incoherently with those relative strengths to give:

$$P_{\alpha\beta}(L=0) \simeq \frac{1}{1 + 3g_A^2} (|\varepsilon_{\beta\alpha}^{udV}|^2 + 3g_A^2 |\varepsilon_{\beta\alpha}^{udA}|^2). \quad (32)$$

Notice that, if only one non-zero  $\varepsilon$  with definite chirality is present, then

$$P_{\alpha\beta}(L=0) \simeq |\varepsilon_{\beta\alpha}^{udP}|^2. \quad (33)$$

Experiment	Channel	Bounds
KARMEN	$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$	$ \varepsilon_{\alpha e}^{\mu e P}  < 0.025,  \varepsilon_{e\mu}^{udA}  < 0.028,  \varepsilon_{e\mu}^{udV}  < 0.059$
NOMAD	$\nu_\mu \rightarrow \nu_\tau$	$ \varepsilon_{\mu\tau}^{udA}  < 0.013,  \varepsilon_{\tau\mu}^{udL}  < 0.013,  \varepsilon_{\tau\mu}^{udR}  < 0.018$
NOMAD	$\nu_e \rightarrow \nu_\tau$	$ \varepsilon_{\alpha\tau}^{\mu e P}  < 0.087,  \varepsilon_{\tau e}^{udL}  < 0.087,  \varepsilon_{\tau e}^{udR}  < 0.12$
NOMAD	$\nu_\mu \rightarrow \nu_e$	$ \varepsilon_{\mu e}^{udA}  < 0.026,  \varepsilon_{e\mu}^{udL}  < 0.026,  \varepsilon_{e\mu}^{udR}  < 0.037$

TABLE I: Bounds (90 % CL) from oscillations at zero distance. In each line, the first bound refers to production NSI, while the other two are for detection NSI.

We will make this assumption when we will summarise the bounds in the last Section. On the other hand, at very high energies, in the deep inelastic scattering regime, the left-handed NSI contribute to the neutrino cross-sections with a strength about twice that of the right-handed, the actual factor being given by the ratio of the neutrino and antineutrino cross sections at high energies for an isoscalar target  $r = \sigma_\nu/\sigma_{\bar{\nu}} \simeq 6.7/3.4 = 1.97$ . We then obtain:

$$P_{\alpha\beta}(L=0) \simeq |\varepsilon_{\beta\alpha}^{udL}|^2 + \frac{1}{r} |\varepsilon_{\beta\alpha}^{udR}|^2. \quad (34)$$

We can therefore use the very precise constraints on flavour oscillations from experiments such as KARMEN [53] and NOMAD [54, 55]. Motivated by the large mass hierarchies and small mixing angles observed in the quark sector, these experiments explored neutrino oscillations at very short baselines with high precision and no evidence of flavour change was found. Both KARMEN and NOMAD produced neutrino beams from  $\pi^+$  decays as well as the subsequent  $\mu^+$  decays and detected them through inverse beta decay. In the case of KARMEN the neutrinos were produced via  $\mu$  decays at rest, so that the neutrino energy was always below 50 MeV. On the other hand, NOMAD aimed at the detection of  $\nu_\tau$ , so higher energies  $\sim 20$  GeV were exploited. Table I contains a summary of the different oscillation channels they explored and the bounds they imply for the NSI parameters.

### III. LOOP BOUNDS

The tree level effects of neutrino NSI are difficult to constrain since neutrino detection and flavour tagging is challenging. However, NSI may mix with four-charged-fermion operators at the loop level inducing flavour-changing charged-lepton interactions, for which

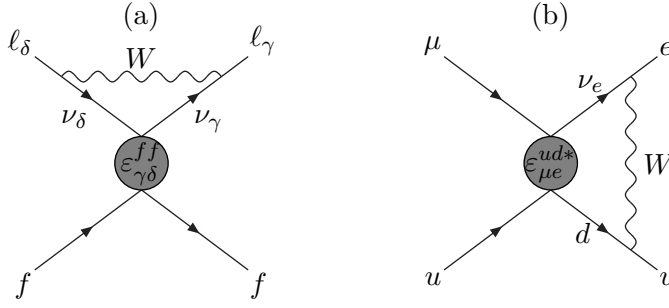


FIG. 1: (a) The vanishing one-loop contribution to the mixing in the running between the matter NSI and the four-charged-fermion operator via  $W$  exchange. (b) The non-vanishing one-loop contribution to the mixing in the running between the charged-current-like NSI and the operator inducing  $\mu \rightarrow e$  conversion in nuclei.

strong bounds exist. In Ref. [34] it was shown that, for a certain class of diagrams (see Fig. 1a), the logarithmic divergences that would indicate the mixing in the running between NSI and four-charged-fermion operators canceled. Therefore, only model-dependent finite contributions remain and no model-independent bound can be derived through one-loop considerations. We have checked that this is also the case for most neutrino NSI at production and detection. There is, however, an exception: the NSI parameter  $\varepsilon_{\mu e}^{udL}$  mixes with the operator that induces muon to electron conversion in nuclei through the diagram of Fig. 1b. The computation of this diagram yields a logarithmic divergence:

$$\frac{3\sqrt{2}G_F\alpha\varepsilon_{\mu e}^{udL}}{2\pi s_w^2} \log\left(\frac{\Lambda}{M_W}\right) [\bar{u}\gamma_\beta P_L u][\bar{\mu}\gamma^\beta P_L e]. \quad (35)$$

Since the coefficient of this divergence can be interpreted as the coefficient of the logarithmic running of this operator, we can estimate the bound by assuming that  $\log(\Lambda/M_W) \simeq 1$ . This gives a contribution to  $\mu \rightarrow e$  conversion in nuclei of the form (see, *e.g.*, Ref. [56]):

$$R(\mu^- \rightarrow e^-) = \frac{m_\mu^5 (2V^{(p)} + V^{(n)})^2 |C|^2}{\Gamma(\mu \text{ capture})}, \quad (36)$$

where  $C$  is the coefficient of the operator in Eq. (35). Using  $R(\mu^- \rightarrow e^-) < 7.0 \cdot 10^{-13}$  for conversion in Au [50] as well as  $V_{\text{Au}}^{(p)} = 0.0974$  and  $V_{\text{Au}}^{(n)} = 0.146$  [56], a very strong bound on the NSI is derived:

$$|\varepsilon_{\mu e}^{udL}| < 1.8 \cdot 10^{-6}. \quad (37)$$

We would like to remark that, also in this case, a quadratic divergence is present. In principle, this contribution could dominate over the logarithmic one, but its value is model-dependent

and reliable bounds cannot be derived from it. The contribution from the logarithmic running could also be canceled, but only at a given scale, which makes the resulting constraint more reliable.

#### IV. NEUTRAL-CURRENT-LIKE NON-STANDARD INTERACTIONS

For completeness, we will now also review the current status of the bounds on NSI matter effects, or neutral-current-like NSI defined in Eq. (1). This type of NSI is the most extensively studied in the literature, since it has been generally assumed that the constraints on the charged-current-like NSI are much stronger. We would like to stress that, in specific models, charged-current-like and neutral-current-like processes are expected with similar strengths [32].

In most phenomenological studies the NSI parameters are reduced to the effective parameters

$$\varepsilon_{\alpha\beta} = \sum_{f,P} \varepsilon_{\alpha\beta}^{fP} \frac{n_f}{n_e}, \quad (38)$$

where  $n_f$  is the number density of the fermion  $f$ . This is the natural parameter in neutrino oscillation analyses since it corresponds to the replacement

$$H_{\text{matter}} = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow V \left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix} \right] \quad (39)$$

in the matter interaction part of the neutrino flavour evolution. Thus, assuming uncorrelated errors, the bounds on  $\varepsilon_{\alpha\beta}$  could be approximated by

$$\varepsilon_{\alpha\beta}^{\oplus} \lesssim \left\{ \sum_P \left[ (\varepsilon_{\alpha\beta}^{eP})^2 + (3\varepsilon_{\alpha\beta}^{uP})^2 + (3\varepsilon_{\alpha\beta}^{dP})^2 \right] \right\}^{1/2} \quad (40)$$

for neutral Earth-like matter with an equal number of neutrons and protons and by

$$\varepsilon_{\alpha\beta}^{\odot} \lesssim \left\{ \sum_P \left[ (\varepsilon_{\alpha\beta}^{eP})^2 + (2\varepsilon_{\alpha\beta}^{uP})^2 + (\varepsilon_{\alpha\beta}^{dP})^2 \right] \right\}^{1/2} \quad (41)$$

for neutral solar-like matter, consisting mostly of electrons and protons. Using the bounds from Refs. [57, 58, 59, 60], but discarding the loop constraints on  $\varepsilon_{e\mu}^{fP}$  [34], the resulting

bounds on the effective NSI parameters would be

$$|\varepsilon_{\alpha\beta}^{\oplus}| < \begin{pmatrix} 4.2 & 0.33 & 3.0 \\ 0.33 & 0.068 & 0.33 \\ 3.0 & 0.33 & 21 \end{pmatrix} \quad \text{and} \quad |\varepsilon_{\alpha\beta}^{\ominus}| < \begin{pmatrix} 2.5 & 0.21 & 1.7 \\ 0.21 & 0.046 & 0.21 \\ 1.7 & 0.21 & 9.0 \end{pmatrix}, \quad (42)$$

respectively. Notice that atmospheric neutrino oscillations also constrain the values of matter NSI through the relation  $\varepsilon_{\tau\tau}^{\oplus} \simeq [|\varepsilon_{e\tau}^{\oplus}|^2 \pm \mathcal{O}(0.1)]/(1 + \varepsilon_{ee}^{\oplus})$  [12, 61]. As long as  $1 + \varepsilon_{ee}^{\oplus}$  is not significantly smaller than one, this would set a stronger bound  $\varepsilon_{\tau\tau}^{\oplus} \lesssim \mathcal{O}(10)$ .

We want to stress the fact that the constraints on  $\varepsilon^e$ ,  $\varepsilon^u$  and  $\varepsilon^d$  have been derived under the assumption of taking one non-zero  $\varepsilon$  at a time. Thus, the approach of combining them together as in Eq. (42) is not fully consistent. For this reason, in the compilation of all the results in the following Section, the bounds will be quoted separately.

## V. SUMMARY OF RESULTS AND CONCLUSIONS

In order to easily overview our results, we here present the constraints from the previous sections in tabularized format. In Tab. II we present the different bounds available for  $\varepsilon_{\alpha\beta}^{\mu e}$  while the bounds for  $\varepsilon_{\alpha\beta}^{ud}$  are presented in Tab. III. Taken all together, the most stringent bounds available for both charged-current-like and neutral-current-like NSI relevant for terrestrial experiments are given by:

$$|\varepsilon_{\alpha\beta}^{\mu e}| < \begin{pmatrix} 0.025 & 0.030 & 0.030 \\ 0.025 & 0.030 & 0.030 \\ 0.025 & 0.030 & 0.030 \end{pmatrix}, \quad (43)$$

$$|\varepsilon_{\alpha\beta}^{ud}| < \begin{pmatrix} 0.041 & 0.025 & 0.041 \\ 1.8 \cdot 10^{-6} & 0.078 & 0.013 \\ 0.026 & 0.013 & 0.13 \\ 0.087 & 0.018 & 0.13 \\ 0.12 & 0.018 & 0.13 \end{pmatrix}, \quad (44)$$

$$|\varepsilon_{\alpha\beta}^e| < \begin{pmatrix} 0.06 & 0.10 & 0.4 \\ 0.14 & 0.10 & 0.27 \\ 0.10 & 0.03 & 0.10 \\ 0.10 & 0.03 & 0.10 \\ 0.4 & 0.10 & 0.16 \\ 0.27 & 0.10 & 0.4 \end{pmatrix}, \quad (45)$$

$\varepsilon_{\alpha\beta}^{\mu e}$	<b>Kin. <math>G_F</math> (<math>L, R</math>)</b>	<b>CKM unit. (<math>V</math>)</b>	<b>Lept. univ. (<math>A</math>)</b>	<b>Oscillation (<math>L, R</math>)</b>
$\varepsilon_{ee}^{\mu e}$	$< 0.030$	$< 0.030$	$< 0.080$	$< 0.025$
$\varepsilon_{e\mu}^{\mu e}$	$(-1.4 \pm 1.4) \cdot 10^{-3}(\mathbb{R}, L)$ $< 0.030$	$< 4 \cdot 10^{-4}(\mathbb{R})$ $< 0.030$	$(-0.4 \pm 3.5) \cdot 10^{-3}(\mathbb{R})$ $< 0.080$	-
$\varepsilon_{e\tau}^{\mu e}$	$< 0.030$	$< 0.030$	$< 0.080$	$< 0.087$
$\varepsilon_{\mu e}^{\mu e}$	$< 0.030$	$< 0.030$	$< 0.080$	$< 0.025$
$\varepsilon_{\mu\mu}^{\mu e}$	$< 0.030$	$< 0.030$	$< 0.080$	-
$\varepsilon_{\mu\tau}^{\mu e}$	$< 0.030$	$< 0.030$	$< 0.080$	$< 0.087$
$\varepsilon_{\tau e}^{\mu e}$	$< 0.030$	$< 0.030$	$< 0.080$	$< 0.025$
$\varepsilon_{\tau\mu}^{\mu e}$	$< 0.030$	$< 0.030$	$< 0.080$	-
$\varepsilon_{\tau\tau}^{\mu e}$	$< 0.030$	$< 0.030$	$< 0.080$	$< 0.087$

TABLE II: Bounds (90 % CL) on the purely leptonic charged-current-like NSI  $\varepsilon_{\alpha\beta}^{\mu e}$ , relevant to the neutrino production through muon decay, *e.g.*, at a Neutrino Factory. The letters  $L, R, V, A$  refer to the chirality of the  $\varepsilon$  which is actually bounded, while  $\mathbb{R}$  stands for the real part of the element only. See the text for details.

$$|\varepsilon_{\alpha\beta}^u| < \begin{pmatrix} 1.0 & 0.05 & 0.5 \\ 0.7 & & \\ 0.05 & 0.003 & 0.05 \\ & 0.008 & \\ 0.5 & 0.05 & 1.4 \\ & & 3 \end{pmatrix}, \quad (46)$$

$$|\varepsilon_{\alpha\beta}^d| < \begin{pmatrix} 0.3 & 0.05 & 0.5 \\ 0.6 & & \\ 0.05 & 0.003 & 0.05 \\ & 0.015 & \\ 0.5 & 0.05 & 1.1 \\ & & 6 \end{pmatrix}. \quad (47)$$

Here, whenever two values are quoted, the upper value refers to left-handed NSI and the lower to right-handed NSI. We would like to stress that, before applying these constraints, the reader should refer to the appropriate Sections in order to be aware of the assumptions under which they were obtained.

To summarise, we have presented the model-independent bounds that can be derived

$\varepsilon_{\alpha\beta}^{ud}$	CKM unit. ( $V$ )	Lept. univ. ( $A$ )	Oscillation	Loop ( $L$ )
$\varepsilon_{ee}^{ud}$	$< 8.6 \cdot 10^{-4}(\mathbb{R})$ $< 0.041$	$(-2.1 \pm 2.6) \cdot 10^{-3}(\mathbb{R})$ $< 0.045$	-	-
$\varepsilon_{e\mu}^{ud}$	$< 0.041$	$< 0.045$	$< 0.028(A)$ $< 0.059(V)$ $< 0.026(L)$ $< 0.037(R)$	-
$\varepsilon_{e\tau}^{ud}$	$< 0.041$	$< 0.045$	-	-
$\varepsilon_{\mu e}^{ud}$	-	$< 0.078$	$< 0.026(A)$	$< 1.8 \cdot 10^{-6}$
$\varepsilon_{\mu\mu}^{ud}$	-	$(2.1 \pm 2.6) \cdot 10^{-3}(\mathbb{R})$ $< 0.078$	-	-
$\varepsilon_{\mu\tau}^{ud}$	-	$< 0.078$	$< 0.013(A)$	-
$\varepsilon_{\tau e}^{ud}$	-	$< 0.13$	$< 0.087(L)$ $< 0.12(R)$	- -
$\varepsilon_{\tau\mu}^{ud}$	-	$< 0.13$	$< 0.013(L)$ $< 0.018(R)$	- -
$\varepsilon_{\tau\tau}^{ud}$	-	$(3.0 \pm 5.5) \cdot 10^{-3}(\mathbb{R})$ $< 0.13$	-	-

TABLE III: Bounds (90 % CL) on the quark charged-current-like NSI  $\varepsilon_{\alpha\beta}^{ud}$ , relevant to the neutrino production through hadron decays as well as detection processes. The letters  $L, R, V, A$  refer to the chirality of the  $\varepsilon$  which is actually bounded, while  $\mathbb{R}$  stands for the real part of the element only. See the text for details.

for various types of NSI. Since the neutral-current-like NSI have been studied extensively in the literature and the bounds on these are fairly well known, we have just summarised these results and concentrated on the charged-current-like NSI, which usually are simply considered to be very strongly bounded, although no model-independent analysis has been readily available. The result of our analysis is that the charged-current-like NSI, which are of interest mostly for their impact on neutrino production and detection, are generally bounded by numbers of  $\mathcal{O}(10^{-2})$ – $\mathcal{O}(10^{-1})$ , except for the very strong loop bound on  $\varepsilon_{\mu e}^{udL}$

due to the operator mixing inducing  $\mu \rightarrow e$  conversion in nuclei. We find that these bounds are about one order of magnitude stronger than the bounds on the neutral-current-like NSI. We therefore argue that production and detection NSI should not be neglected with respect to matter NSI, especially taking into account that, in most realisations, both kinds of NSI are induced with similar strengths. Moreover, NSI saturating the bounds derived here will be within the sensitivity reach of planned neutrino oscillation experiments. However, as discussed in the introduction, most models leading to NSI generally affect other processes and therefore stronger bounds than the ones derived here apply.

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